ENERGY BALANCE OF AVERAGED AND PULSATING MOTION IN ANNULAR TURBULENT FLOWS

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UDC 532.542:532.517.4

The energy balance of averaged and pulsating motion in the annular channels between coaxial rotating cylinders is considered.

The results of an experimental investigation into hydrodynamics and heat transfer in annular turbulent flows between coaxial rotating cylinders were presented in [1-5]. A fuller representation of the laws of rotational flow may be obtained on considering the energy balance of averaged and pulsating motion.

The energy balance equation of averaged motion $E_0 = 0.5\,(\bar{v}_x^2 + \bar{v}_r^2 + \bar{v}_\varphi^2)$ may be written in the following dimensionless form, in the absence of external forces and on the assumption of constant physical properties of the liquid:

$$\underset{A}{\frac{\partial}{\partial \bar{t}}(E_0/v_{*i}^2)} - \frac{\bar{\varphi}_i}{\mathrm{Re}_i}\underset{B}{\left[\frac{d}{d\bar{r}}\left(\frac{d\bar{\varphi}_i}{d\bar{r}} - \frac{\bar{\varphi}_i}{\bar{r}}\right) + \frac{2}{\bar{r}}\left(\frac{d\bar{\varphi}_i}{d\bar{r}} - \frac{\bar{\varphi}_i}{\bar{r}}\right)\right]} + \underset{C}{\frac{1}{\bar{r}}\frac{d}{d\bar{r}}\left(\bar{r}\bar{\varphi}_i\frac{\overline{v_r'v_\varphi'}}{v_{*i}^2}\right)} - \underset{D}{\frac{\overline{v_r'v_\varphi'}}{v_{*i}^2}\left(\frac{d\bar{\varphi}_i}{d\bar{r}} - \frac{\bar{\varphi}_i}{\bar{r}}\right)} = 0. \quad (1)$$

This relates to the plane annular turbulent flow of an incompressible liquid, statistically homogeneous on cylindrical surfaces corresponding to constant radius. In flows of this kind the conditions $\bar{v}_x = \bar{v}_r = 0$, $\bar{v}_\varphi = \bar{v}_\varphi(\bar{r})$ and $\bar{p} = \bar{p}(\bar{r})$ are satisfied for the velocity components and pressure, while the derivatives with respect to the axial (x) and tangential (φ) coordinates of the averaged quantities equal zero.

The first term of this equation (A) has the physical sense of a local change in the kinetic energy of the averaged motion, the second (B) and third (C) terms correspond to the work of viscous and turbulent shear stresses, while the last one (D) describes mutual transformations of the energy of averaged and pulsating motion [6, 7].

The turbulent energy balance equation $E = 0.5\,(\overline{v_x'^2} + \overline{v_r'^2} + \overline{v_\varphi'^2})$ for the flow under consideration takes the form

$$\underset{I}{\frac{\partial}{\partial \bar{t}}(E/v_{*i}^2)} + \underset{II}{\frac{\overline{v_r'v_\varphi'}}{v_{*i}^2}\left(\frac{d\bar{\varphi}_i}{d\bar{r}} - \frac{\bar{\varphi}_i}{\bar{r}}\right)} - \frac{1}{\mathrm{Re}_i}\underset{III}{\left[\overline{\left(\frac{\partial v_x'/v_{*i}}{\partial \bar{x}}\right)^2} + \overline{\left(\frac{\partial v_x'/v_{*i}}{\partial \bar{r}}\right)^2} + \overline{\left(\frac{\partial v_x'/v_{*i}}{\bar{r}\partial\varphi}\right)^2} + \overline{\left(\frac{\partial v_r'/v_{*i}}{\partial \bar{x}}\right)^2} + \overline{\left(\frac{\partial v_r'/v_{*i}}{\partial \bar{r}}\right)^2} + \overline{\left(\frac{\partial v_r'/v_{*i}}{\bar{r}\partial\varphi}\right)^2} + \overline{\left(\frac{\partial v_\varphi'/v_{*i}}{\partial \bar{x}}\right)^2} + \overline{\left(\frac{\partial v_\varphi'/v_{*i}}{\partial \bar{r}}\right)^2} + \overline{\left(\frac{\partial v_\varphi'/v_{*i}}{\bar{r}\partial\varphi}\right)^2}\right]} = 0. \quad (2)$$

In Eqs. (1) and (2) the notation $\bar{\varphi}_i = v_\varphi/v_{*i}$; $v_{*i} = \sqrt{\tau_i/\rho}$; $\bar{x} = x/(r_2 - r_1)$; $\bar{r} = r/(r_2 - r_1)$; $\bar{t} = tv_{*i}/(r_2 - r_1)$; $\mathrm{Re}_i = v_{*i}(r_2 - r_1)/\nu$, i = 1, 2 corresponds to the rotation of the internal and external cylinder, respectively.

Kazan Power Institute, Alma Ata. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 18, No. 4, pp. 664-670, April, 1970. Original article submitted August 12, 1968.

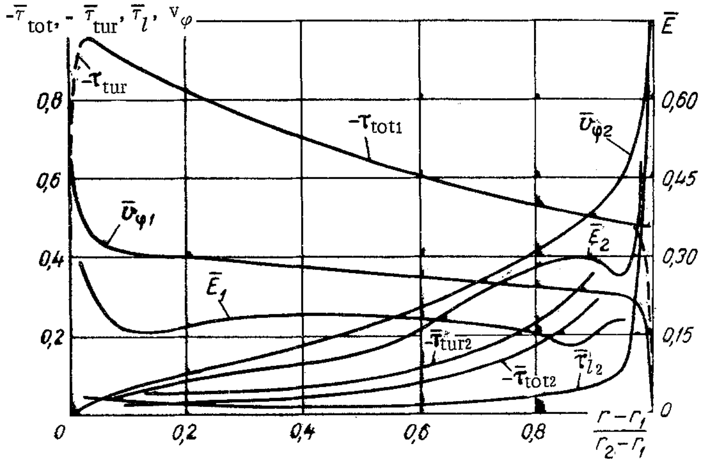

Fig. 1. Distribution of the total ($\bar{\tau}_{tot}$), turbulent ($\bar{\tau}_{tur}$), and laminar ($\bar{\tau}_l$) tangential frictional force, the average velocity (v_φ), and the turbulence energy ($\bar{E}$) in channels with internal and external rotating cylinders respectively.

In Eq. (2) we have omitted the small terms associated with the convective energy transfer due to the turbulence of the average motion and also with the related viscous and turbulent diffusion.

The viscous and turbulent diffusion of the energy of the pulsating motion only play an appreciable part in the flow close to the wall, in the laminar and transitional layer [7].

The first term (I) of Eq. (2) represents the local change in the kinetic energy of the turbulence. The second term (II) differs from the fourth term (D) in Eq. (1) in sign only, and as before describes mutual transformations of the energy of averaged and pulsating motion; the third term (III) represents the viscous dissipation of the pulsating energy into heat.

Thus in the approximation under consideration the principal role in the energy balance of the pulsating motion is played by the generation (II) and dissipation (III) of energy. For steady-state flow ($dE/dt = 0$) these quantities are approximately the same over a great part of the channel cross section, so that the turbulence is practically in a state of energy equilibrium. In this case, by determining the term (D) experimentally we may find the energy dissipation (III).

Figure 1 shows the distribution of the average velocity ($\bar{v}_\varphi = v_\varphi / v_{\varphi i}$), the turbulence energy ($\bar{E} = E/u_{*i}^2$), and the tangential frictional stress, comprising the total ($\bar{\tau}_{tot} = \tau_{tot}/\tau_i$; ($\tau_{tot} = \tau_l + \tau_{tur}$), turbulent ($\bar{\tau}_{tur} = \tau_{tur}/\tau_i$; $\tau_{tur} = -\rho\overline{v'_\varphi v'_r}$), and laminar [$\bar{\tau}_l = \tau_l/\tau_i$; $\tau_l = \mu(dv_\varphi/dr - v_\varphi/r)$] components in channels with internal and external rotating cylinders respectively [3, 5]. Using these data we may readily calculate all the terms in the balance equation for the average and pulsating energy.

In the calculation, the correlation ($\overline{v'_r v'_\varphi}$) close to the walls of the annular channel was found from the expression $\overline{v'_r v'_\varphi} = -r/\rho + \nu(d\bar{v}_\varphi/dr - \bar{v}_\varphi/r)$, while $\tau/\rho = -v_{*1}^2 r_1^2/r^2$ in the case of the rotation of the inner cylinder.†

The results of the calculations relating to the rotation of the inner cylinder are presented in Figs. 2 and 3.

It should be noted, first of all, that the energy balance of the averaged and pulsating motion holds to a fair accuracy over practically the whole radial cross section of the channel.

However, the values of the terms in Eq. (1) vary with radius.

The inner rotating wall of the annular channel sets the neighboring layer of liquid in motion by virtue of adhesion. As a result of the operation of viscous shear stresses, there is an increment in the kinetic energy of averaged motion in this region of flow (curve B in Fig. 2a).

† Here and subsequently the index 1 refers to the inner and the index 2 to the outer cylinders.

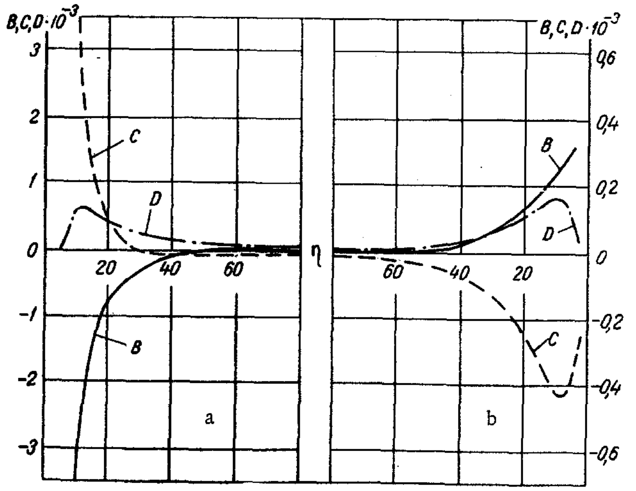

Fig. 2

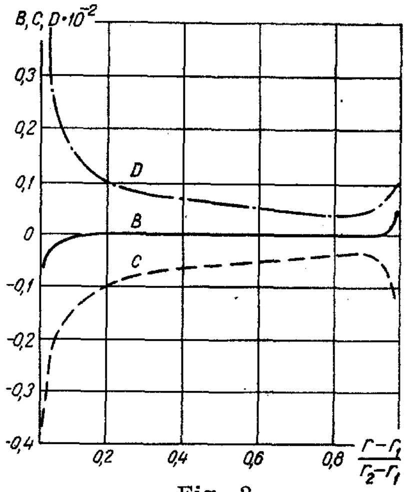

Fig. 3

Fig. 2. Energy balance close to the walls of the annular channel: a) at the inner rotating surface; b) at the outer stationary surface.

Fig. 3. Energy balance in the central region of the channel with an international rotating cylinder.

The averaged flow is retarded under the influence of turbulent shear stresses, losing a certain proportion of kinetic energy (Fig. 2a). The other part of the kinetic energy of the average motion is converted into energy of pulsating motion (generation of turbulence). Owing to the generation of turbulence energy, the losses in the energy of averaged motion increase sharply in the direction of the wall (curve D).

In the boundary region of the flow (approximately $30 < \eta < 80$ or $(r - r_1)/(r_2 - r_1) < 0.2$) the generation of turbulence (curve D) exceeds the inflow of energy directly due to work of the viscous shear stresses (curve B). The balance is made up by an additional inflow of energy due to the work of the turbulent shear stresses (C).

Thus a flow of kinetic energy associated with the averaged motion is directed into this region of flow, and is converted into turbulence energy in the latter.

On moving away from the rotating wall, the component of the energy balance characterizing the increment in the kinetic energy of averaged motion due to the work of the viscous frictional forces falls rapidly, and for $(r - r_1)/(r_2 - r_1) > 0.2$ it becomes negligibly small (curve B in Fig. 2a). The energy losses associated with the generation of turbulence (curve D) are compensated by an increment due to the action of turbulent shear stresses (curve C). This component actually passes through zero and for $\eta > 30$ becomes negative, which corresponds to an increase in kinetic energy.

Near the outer stationary wall of the channel (r_2) the role of the components B, C, and D of the averaged-motion energy balance is very different from their role close to the rotating wall.

The increment in kinetic energy is here effected by the action of the turbulent shear stresses (curve C in Fig. 2b). This increment is compensated by a loss in the kinetic energy of averaged motion, owing to its conversion into turbulence energy (curve D) and work of viscosity friction (curve B). The turbulence energy generated by the averaged motion is dissipated into heat by virtue of the viscous forces.

The generation and dissipation of the energy of pulsating motion practically balance each other over the greater part of the width of the annular gap (curves C and D in Fig. 3).

Thus in the direction from the rotating wall of the channel to the stationary wall there is a flow of kinetic energy of the averaged motion; near the rotating wall this is associated with the work of viscous frictional forces, and over the remaining, greater part of the cross section with the work of turbulent shear stresses. This flow is balanced close to the rotating wall by the energy loss due to the work of the turbulent shear stress and the generation of turbulence, and close to the stationary surface by the viscous shear stress and the generation of turbulence energy.

In the central region (region of developed turbulent flow) the flow of kinetic energy associated with the averaged motion is compensated by transformation into turbulent energy. The maximum generation of pulsational energy occurs near the walls for $\eta \approx 10$. In the greater part of the annular-channel cross section the generated turbulence energy is directly converted into heat by turbulent dissipation.

It would appear that, as in the case of rectilinear flows [7], a more complicated picture of the distribution of pulsating energy should occur in the immediate vicinity of the walls (in the region of the laminar and transitional layer), where a substantial part should be played by the terms associated with the viscous and turbulent diffusion of turbulence energy. The turbulent diffusion in particular, ensures the transfer of turbulence energy from the region of its maximum generation to the inner and outer layers of the liquid.

A considerably more complex distribution of the energy balance occurs in the case of annular flow with stable stratification, created by the rotation of the outer cylinder, with the inner cylinder stationary. The action of the centrifugal forces here leads to a sharp quenching of the intensity of the pulsating motion, and hence to a reduction in the hydrodynamic resistance and heat transfer. In view of this, the turbulent exchange processes are affected to a considerable, often decisive, extent by various external actions operating on the flow, such as eddy currents at the end surfaces of the channel, the turbulizing flow in the measuring device [4], vibration or roughness of the rotating and stationary walls of the channel, etc.

End effects have a particularly marked influence [1, 4]. The presence of a stationary end wall leads (owing to the retardation of the neighboring layers of liquid and the action of the radial pressure field) to the development of a secondary vortical motion, directed close to the wall in the direction of the stationary (inner) cylinder and carrying elements of liquid rich in energy. The flow takes on a complex three-dimensional character.

Exclusion of the effects of the end walls and other external actions laminizes the flow for the angular velocities of the outer cylinder usually attained in practical experiments [1].

The disruption of the plane flow due to the development of secondary eddy currents at the ends of the channel prevents us from estimating the energy balance of turbulent motion on the basis of Eqs. (1) and (2).

However, the use of the more rigorous equations, containing terms associated with the convective transfer of the average motion and with viscous and turbulent diffusion, is hardly feasible at the present time owing to the absence of adequate experimental data, and this will have to be a subject for later study.

Figure 1 shows the distribution of the kinetic energy of pulsating motion $\overline{E}_2 = E/v_{*2}^2$ and the correlation $\overline{\tau}_{tur2} = v_r' v_\varphi' / v_{*2}$ measured in an annular channel with an outer rotating cylinder [4]. We see from the figure that the energy $\overline{E}_2$ falls rapidly in the direction of the inner stationary wall. The correlation $v_r' v_\varphi' = \tau_{tur2}/\rho$ behaves in an analogous way.

It should be noted that the correlation $(\overline{v_r' v_\varphi'})$ has a positive value in the region of the measurements. This is primarily due to the mixing of the energy-rich layers of liquid near the rotating cylinder and low-energy layers near the stationary one by the secondary eddy currents at the end walls. In addition to this, further mixing is provided by the measuring devices, vibration, surface roughness, etc. The action of the field of centrifugal forces under conditions of stable stratification amounts to the fact that liquid elements possessing higher tangential velocities tend to move in the direction of the outer rotating wall, and those with lower tangential velocities toward the stationary wall.

Thus statistically the mean value of the correlation $\overline{v_r' v_\varphi'}$ becomes positive. A similar qualitative picture holds true in the centrifuging of a liquid inhomogeneous in density, when the lighter elements of the liquid assemble close to the axis of the centrifuge.

The difference in the signs of the turbulent $(\tau_{tur2} = -\rho\overline{v_r' v_\varphi'})$ and viscous $(\tau_{l2} = \mu(d\overline{v}_\varphi/dr - \overline{v}_\varphi/r))$ shear stresses leads to the following curious fact. The total shear stress found from the equation $\tau_{tot2} = \tau_{tur} + \tau_l = -\rho\overline{v_r' v_\varphi'} + \mu(d\overline{v}_\varphi/dr - \overline{v}_\varphi/r)$ may have a sign opposite to that of the frictional stress at the rotating wall of the channel (Fig. 1). This may occur because the principal mechanism determining the transfer of the torsional stress to the inner (stationary) cylinder is evidently transfer by the secondary eddy currents arising at the end walls of the channel. The sign difference between τ_{tur2} and τ_{l2} may also be explained by the reduction in the moment of the frictional forces at the inner cylinder (to values smaller than those associated with laminar flow) observed in experiments involving flow in a channel with an outer rotating cylinder [4]. It should be noted, at the same time, that the end effects do not exert any substantial influence on the processes of turbulent transfer in channels with an inner rotating cylinder (with unstable stratification of the flow) [3].

NOTATION

E_0	is the energy of averaged motion;
E	is the energy of turbulent motion;
$\bar{v}_x$, $\bar{v}_r$, $\bar{v}_\varphi$	are the axial, radial, and tangential components of the velocity vector;
$\bar{p}$	is the pressure;
x, r	are the axial and radial coordinates;
t	is the time;
v_*	is the dynamic velocity;
τ_{tot}	is the total tangential frictional stress;
$\bar{\tau}_{tot} = \bar{\tau}_{tur} + \bar{\tau}_l$;	
τ_{tur}	is the turbulent friction;
$\tau_{tur} = -\rho\overline{v'_\varphi v'_r}$;	
τ_l	is the viscous friction;
$\tau_l = \mu(dv_\varphi/dr - v_\varphi/r)$;	
ρ	is the density;
ν	is the kinematic viscosity;
$\bar{v}'_x$, $\bar{v}'_r$, $\bar{v}'_\varphi$	are the axial, radial, and tangential components of the mean-square pulsations of the velocity vector;
φ	is the dimensionless tangential velocity;
η	is the universal coordinate.

LITERATURE CITED

1. F. Wendt, "Turbulent flow between two rotating coaxial cylinders," Ing. Arch., 4 (1933).
2. K. M. Becker and J. Kaye, Teploperedacha, No. 2 (1962) (Russian translation of the Transactions of the American Society of Mechanical Engineers).
3. B. P. Ustimenko and V. N. Zmeikov, Teplofiz. Vys. Temp., 1, No. 2 (1964).
4. V. N. Zmeikov and B. P. Ustimenko, in: Applied Heat Physics [in Russian], Nauka, Alma-Ata (1965).
5. B. P. Ustimenko and V. N. Zmeikov, Teplofiz. Vys. Temp., 5, No. 4 (1967).
6. A. L. Townsend, Structure of Turbulent Flow with Transverse Shear [Russian translation], IL (1959).
7. I. O. Khintse, Turbulence [in Russian], Fizmatgiz, Moscow (1963).